NASA and IEEE 754 Format

Differences

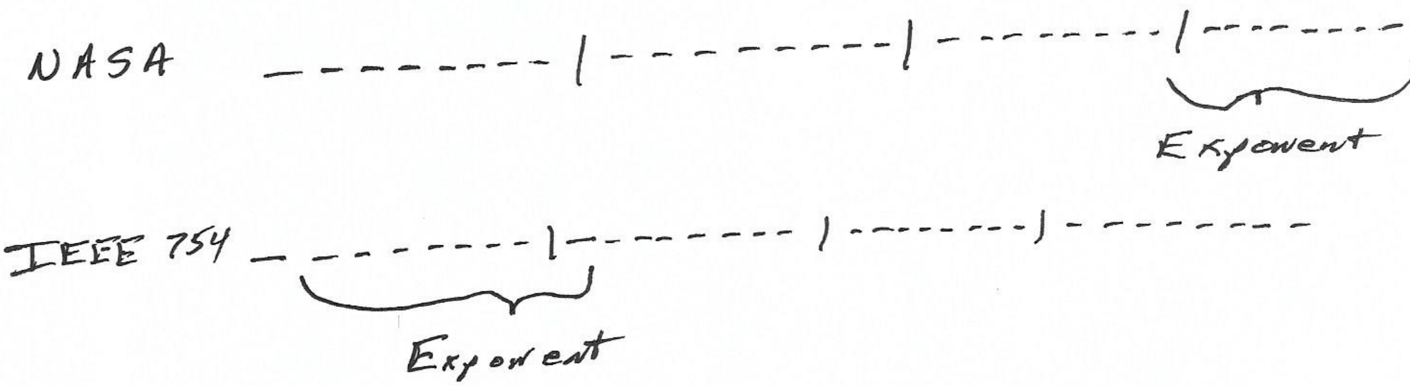
1) Scaling

NASA → $0.1XXX \times 2^{E}$

IEEE → $1.XXX \times 2^{E-1}$

The exponent has been shifted

2) The position of the Exponent is shifted. Moved to front Bits.



3) The exponent is biased by adding 127. No 2's complement

Nasa → $0.1xxx \times 2^{E}$

Vs. $1.xxx \times 2^{E-1+127}$

4) Lastly the 1 is dropped

NASA → $0.1xxx \times 2^{E}$

vs. $.xxx \times 2^{E-1+127}$

Gains an extra Bit of accuracy.

So, Let's start with an example

0.1_{10} → First convert to base 16

$$0.1 \times 16 = 1.\underline{6}$$
$$0.6 \times 16 = 9.\underline{6}$$
$$0.6 \times 16 = 9.\underline{6}$$
$$\vdots$$

$$0.1_{10} = 0.19999\ldots_{16} = 0.1\underline{9}_{16}$$

Let's have fun and add this infinite sequence to make sure it is 0.1_{10}

$$\sum_{i=0}^{\infty} c^{i} = \frac{1}{1-c}, \quad c < 1$$

Let's prove the previous formula

$$S_N = c^0 + c^1 + c^2 + c^3 + \dots c^N$$

$$= \sum_{i=0}^{N} c^i$$

No multiply by c

$$cS_N = c\sum_{i=0}^{N} c^i = c^1 + c^2 + c^3 + \dots c^{N+1}$$

Now subtract the first from the second

$$S_N = c^0 + c^1 + c^2 + \dots c^N$$

$$-cS_N = c^1 + c^2 + \dots c^N + c^{N+1}$$

$$(1-c)S_N = c^0 \quad + c^{N+1}$$

$$S_N = \frac{c^0 + c^{N+1}}{1-c}$$

As $N >>$ $c^{N+1} = 0$

$$\lim_{N \to \infty} S_N = \frac{1}{1-c}$$

If we use ths formula then we can show equivalency

$$0.1\underline{9}_{16} = 1/16 + 9/16^2 + 9/16^3 + 9/16^4 + \dots$$

$$= 1/16 + 9/16^2 \sum_{i=0}^{\infty} (1/16)^i$$

$$= 1/16 + 9/16^2 (1 + 1/16 + 1/16^2 + \dots)$$

↓ use formula

$$= 1/16 + 9/16^2 \left(\frac{1}{1 - 1/16}\right)$$

$$= 1/16 + 9/16^2 (16/15)$$

$$= 1/16 + 9/16 \cdot 15$$

$$= \frac{15 + 9}{16 \cdot 15} = \frac{24}{240} = \underline{\underline{.1_{10}}}$$

So $0.1_{10} = 0.1\underline{9}_{16}$

Now put this into NASA format

$$0.19_{16} = 0.0001\ 1001\ 1001\ 1001 \ldots$$

$$= 0.1100\ \underline{1100} \times 2^{-3}$$

$$= 0.\underline{1100} \times 2^{-3} \rightarrow$$

2's complement

```
  0000 0011
  1111 1100
        + 1
  ---------
  1111 1101
```

0.1100110|0110011|0|0110011|0|11111101

6 6 6 6 6 6 F D

This would be NASA format

Now convert to IEEE 754

$0.\underline{1100} \times 2^{-3}$ ⟸ NASA

$= 1.\underline{1001} \times 2^{-4}$

127 − 4 = 123

0 | 0 1 1 1 1 0 1 | 1 | 1 0 0 1 1 0 0 | 1 1 0 0 1 1 0 0 | 1 1 0 0 1 1 0 0

↑ 1

3 D C C C C C C

Rounding Up

3 D C C C C C D